The Influence of Color Interreflections on Lighting Simulations

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Abstract

Most lighting simulation programs represent color as RGB triplets in a device-dependent color space such as ITU-R BT.709. Implicit in this representation is the assumption that interreflections between colored surfaces can be accurately calculated using three separate color bands. We demonstrate that while this assumption generally holds for most architectural finishes, it can result in substantial prediction errors for saturated colors.

Introduction

Here is a simple question with a perhaps surprising answer. Suppose we have two identical rooms. One room is painted dark grey with a reflectance of 20 percent; the other room is painted red with a reflectance of 20 percent. The horizontal illuminance at a point on the floor in the grey room is 250 lux; what is the horizontal illuminance at the same point in the red room?

Most professional lighting designers will quickly answer that it is obviously the same – 250 lux. The correct answer however is that it depends on the room geometry. More subtly, it also depends on the spectral power distribution (SPD) of the light source and the spectral reflectance distribution (SRD) of the painted surfaces.

Given that most lighting simulation programs do not consider the spectral properties of light sources or materials, we have to ask the question: can we accurately calculate horizontal illuminance (and other photometric quantities) with these programs when color is involved?

Sumpner's Principle

Imagine an infinitely large room with an infinite array of direct lighting. (We can think of this as a luminous ceiling.) The light emitted by the luminaires will directly illuminate the floor. Of this light, 20 percent will be reflected upwards to the ceiling, while the rest will be absorbed. The ceiling will reflect 20 percent of the indirect light towards the floor, which will reflect 20 percent back towards the ceiling and so on, until all of the light is absorbed.

Calculating the total illuminance E_T of the floor is simple. Given a surface reflectance r and direct illuminance E_D due to the luminaires, it is:

$$E_T = (1 + r + r^2 + r^3 + \cdots) * E_D = E_D/(1 - r) \quad (1)$$

In mathematical terms, this is a MacLaurin series expansion for $0 \leq r < 1$. It is also the basis of Sumpner's Principle (Cuttle 1991), which estimates mean room surface illuminance. (This equation does however accurately describe the surface illuminance of an integrating sphere.)

For our example of $r = 0.20$, this becomes $E_T = 1.25 * E_D$. In other words, 25 percent of the total illuminance of the floor is due to interreflected light between the floor and the ceiling. As the reflectance r is increased, more and more of the total illuminance E_T is due to light interreflected from the room surfaces.

This begs the question, however. We refer to both the grey and red surfaces as having reflectances of 20 percent, but what does this really mean?

Reflectance and Color

ANSI/IES RP-16 (IES 2010) defines the reflectance of a surface as "the ratio of the reflected flux to the incident flux," where flux may be either radiant flux (measured in watts) or luminous flux (measured in lumens). This seems simple enough, but we also have the definition of radiant flux:

$$\Phi_e = \int \Phi_{e,\lambda}\, d\lambda \quad (2)$$

and luminous flux: as:

$$\Phi_V = K_m \int_{380}^{780} \Phi_{e,\lambda} V(\lambda) d\lambda \quad (3)$$

in terms of spectral radiant flux $\Phi_{e,\lambda}$ (measured in watts per nanometer), where $K_m = 683$ lumens per watt for photopic vision.

What this implicitly says is important:

The reflectance of a colored surface depends on the spectral power distribution of the illuminant.

As an example, suppose we measure the reflectance of a red surface with the spectral reflectance distribution shown in Figure 1 as being 20 percent when illuminated by D_{65} daylight. If we illuminate the same surface with quasimonochromatic blue light from a 465 nm LED however, the measured reflectance is less than one percent.

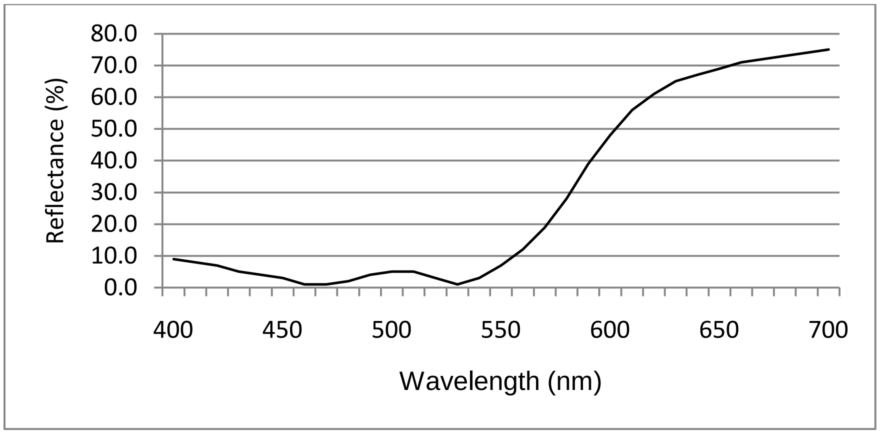


Figure 1 – Spectral reflectance distribution of red material (20 percent reflectance with D_{65} illuminant)

Equation 1 assumes that the reflectance *r* is constant for each reflection. This however is true only for grey surfaces. Each reflection from a colored surface modifies the SPD of the incident light. In colloquial terms, the light reflected from a red surface becomes redder with each subsequent reflection.

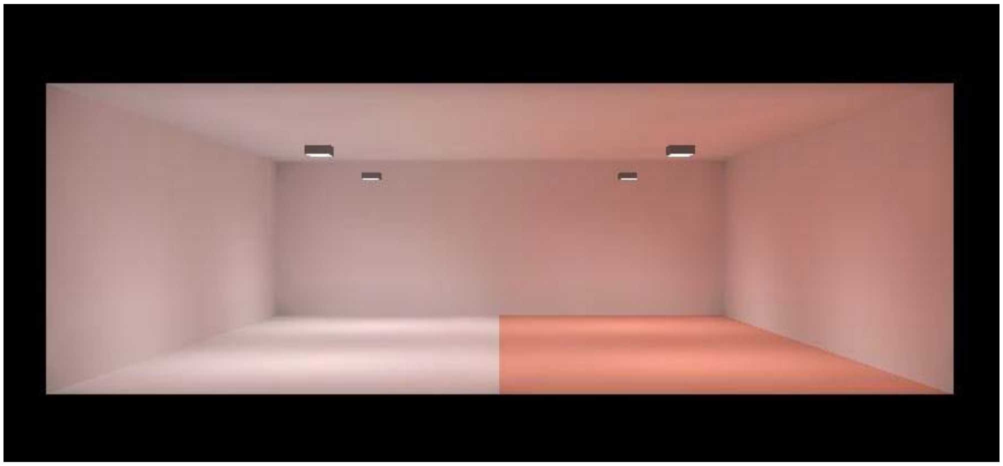


Figure 2 – Color interreflections example.

Equation 1 therefore needs to be generalized to:

$$E_T = K_m \int_{380}^{780} \left(E_{D,\lambda}/(1 - r_\lambda)\right) V(\lambda) d\lambda \tag{4}$$

for colored surfaces, where $E_{D,\lambda}$ is the spectral direct irradiance.

It is thus evident that the horizontal illuminance of the red room will be less than that of the grey room with a D_{65} illuminant. We now have to address the issue of calculating these values.

Calculating Color

Professional lighting designers think strictly in terms of "white light" when performing lighting calculations, with no thought given to the SPD Φ_λ of the natural or electric light sources. This is unfortunately all but mandated by their lighting simulation programs, which model color as RGB triplets.

ITU Recommendation BT.709 (ITU 2002) specifies the CIE 1931 *xy* chromaticities of the primary colors and D_{65} white point of high-definition television monitors and computer displays as:

Color	*x*	*y*
Red	0.640	0.330
Green	0.300	0.600
Blue	0.150	0.060
White (D_{65})	0.3127	0.3290

where the white point is defined as the chromaticity generated by additively mixing the three primary colors at maximum intensity. Lighting simulation programs therefore enable users to specify material and light source colors as BT.709 red-green-blue (RGB) triplets or equivalent CIE 1931 *xyY* color space values according to the transformation:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = k \begin{bmatrix} 3.2405 & 1.5371 & -0.4985 \\ -0.9693 & 1.876 & 0.0416 \\ 0.0556 & -0.2040 & 1.0572 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{5}$$

where k is a scaling constant.

By default, lighting simulation programs assume that all "white" light sources have a correlated color temperature (CCT) of 6500 Kelvin. They further perform their calculations for direct and interreflected light using three color bands: red, green, and blue.

Referring to Equation 3, this becomes:

$$E_T = K_m \left(E_{D,R}/(1 - r_R) + E_{D,G}/(1 - r_G) + E_{D,R}/(1 - r_B)\right) \tag{6}$$

where the subscripts R, G, and B indicate the respective values for the three color bands.

This represents a very coarse discretization of the SPD and SRD. The question is whether this discretization results in significant errors in calculating photometric quantities for architectural environments.

We can estimate these errors by increasing the resolution of the SPD discretization and using:

$$E_T = K_m \sum \left(E_{D,\lambda}/(1 - r_\lambda)\right) V(\lambda) \Delta\lambda \tag{7}$$

for $\Delta\lambda = 10$ nm over the visible spectrum. We also however need to know or estimate the SRDs of common architectural materials so that we can discretize them as well.

Spectral Reconstruction

There is a large body of multispectral imaging literature devoted to the reconstruction of SRDs from sparsely sampled spectra. While the techniques vary, the basic idea is to measure the SRDs of many different materials with a high spectral resolution and store a representation of them in a database. Given the sparsely sampled SRD of an unknown material, its high resolution SRD can be interpolated from the

SRDs in the database whose sparsely sampled values most closely match those of the unknown material.

Fairman and Brill (2004) presented a technique based on principal components analysis (PCA) of 3,534 measured SRDs for color samples from the Munsell Book of Color, the Swedish Natural Color System and the OSA-UCS color atlas. Given any set of ITU-R BT.709 RGB triplets whose chromaticity is within the intersection of the color gamuts of the BT.709 primaries and these color samples, a reasonably accurate SRD can be reconstructed for any $\Delta\lambda$ using only the mean and first three principal components of the dataset for an assumed illuminant (such as CIE D65).

The mean and first three principal components of Fairman and Brill's dataset (Figures 5 and 6) may appear remarkably smooth in comparison to the illuminant SPDs, but they are consistent with numerous other studies. Maloney (1986) noted that there are strong molecular constraints on the variability of SRDs for natural materials (including wood, paper, minerals, and biological pigments), while Westland et al. (2000) reported a band limit for natural and man-made surfaces of approximately 0.015 to 0.020 cycles per nanometer. They further reported (again consistent with other studies) that 96 percent of SRD variance for natural materials and 98 percent for man-made materials can be accounted for by a linear model with only 3 parameters.

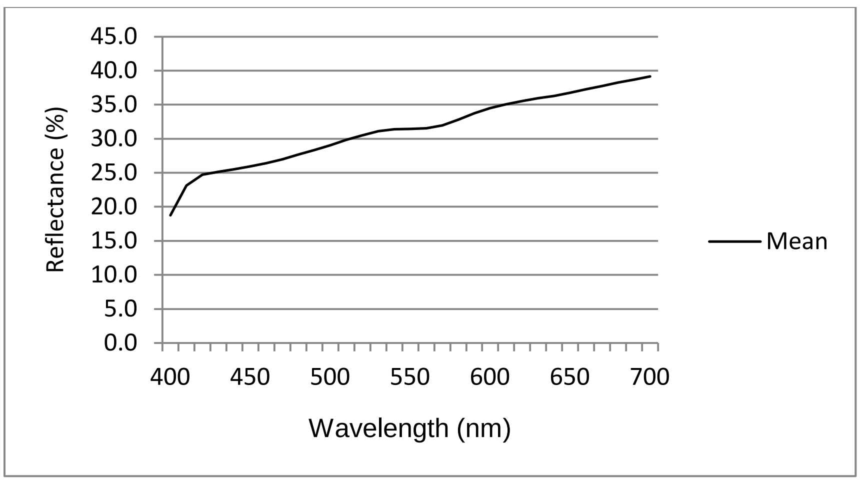


Figure 5 - Mean of 3,534 measured SRDs (Fairman and Brill 2004)

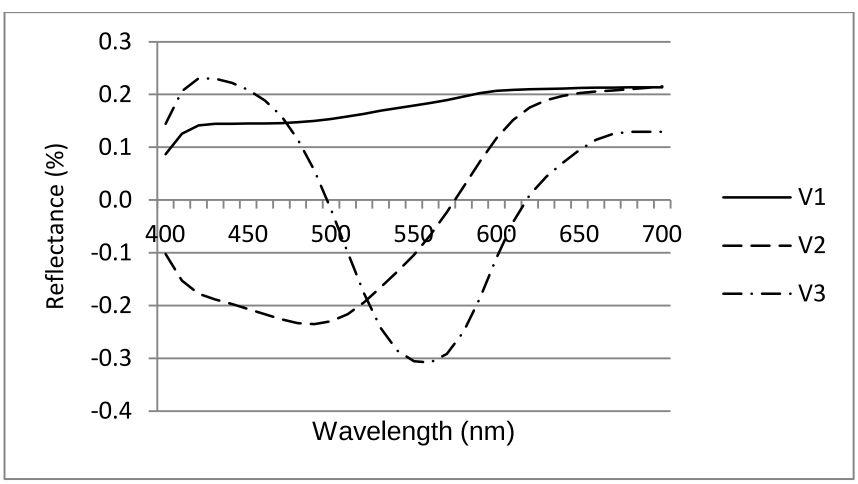


Figure 6 – First three principal components of 3,534 measured SRDs (Fairman and Brill 2004)

We do not however need to reconstruct SRDs as accurately as possible. Instead, we only need reasonable approximations for common architectural materials when given their CIE *xyY* or ITU-R BT.709

color space values. We further need a spectral reconstruction algorithm that is based on a large measured sample of architectural materials, and which generates continuously variable SRDs over the color gamut of ITU-R BT.709. Fairman and Brill's technique meets these requirements.

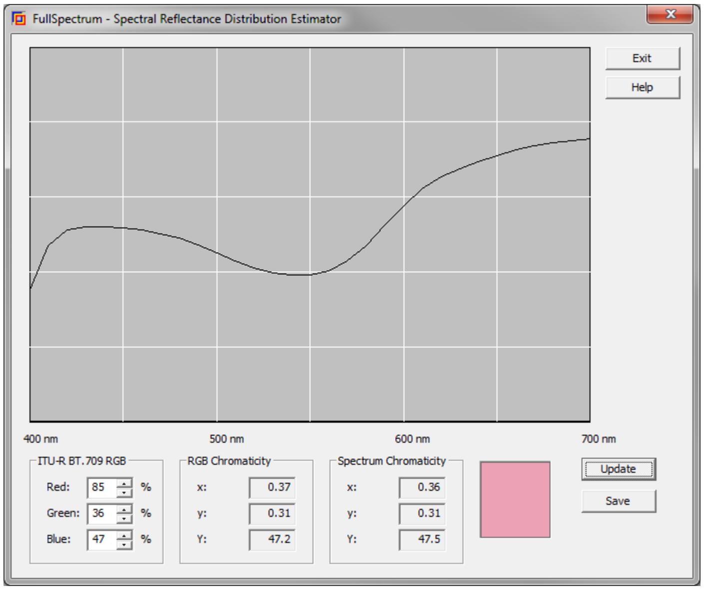


Figure 7 – Implementation of Fairman and Brill (2004) spectral reconstruction technique

Error Analysis

Given that most lighting simulation programs enable the user to choose material colors in the ITU-R BT.709 color space with red, green, and blue values ranging from 0.0 to 1.0, it makes sense to divide the RGB_{709} color cube into ten equally-spaced steps in each dimension and perform the following:

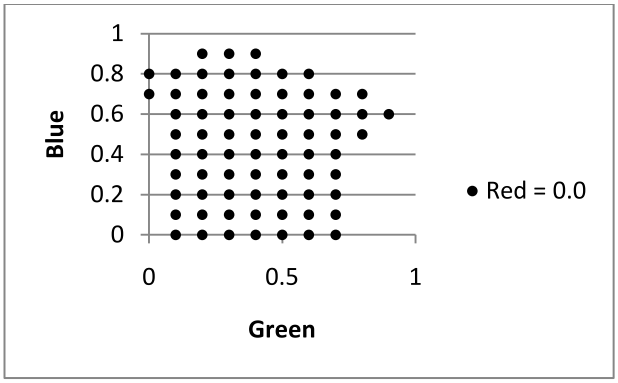
1. Convert RGB_{709} values to CIE XYZ tristimulus values
2. Calculate interreflected RGB_{709} values (Equation 6)
3. Calculate RGB_{709} luminance L_{709}
4. Calculate reflectance spectrum according to Fairman and Brill (2004)
5. Calculate interreflected reflectance spectrum values (Equation 7)
6. Calculate reflectance spectrum luminance L_{RS}
7. Calculate error $\varepsilon = L_{RS} / L_{709}$

The RGB_{709} luminance value is given by:

$$L_{709} = 0.2026 * R + 0.7152 * G + 0.0722 * B \quad (8)$$

where the parameters are the middle row of the inverse of the XYZ-to-RGB transformation matrix (Equation 5). Similarly, the reflectance spectrum luminance L_{RS} is calculated by multiplying the reflectance spectrum by the photopic luminous efficiency function $V(\lambda)$.

Figure 8 shows the range of RGB_{709} values (scaled to the range of 0.0 to 1.0) over which the error ε is between 0.80 and 1.25. Various studies have shown that the accuracy of lighting calculations is approximately ±10 percent for electric lighting and ±20 percent for daylighting, so these are reasonable error limits.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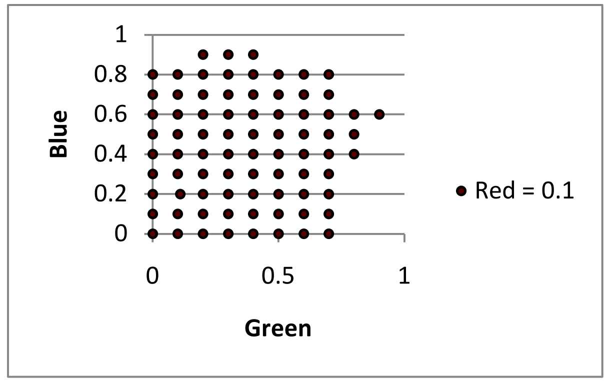
0.8
0.6
0.4
0.2
0
0
0.5
1
Green
Red = 0.0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0.5
1
Green
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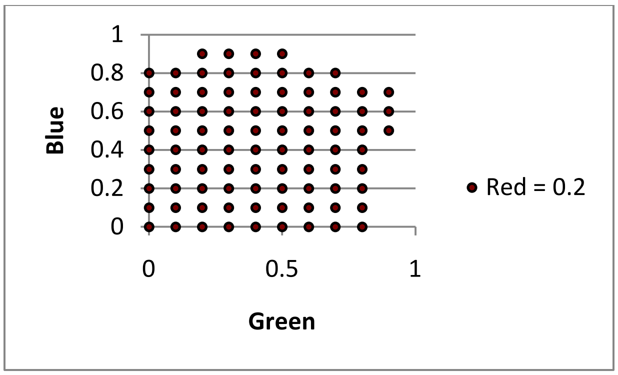
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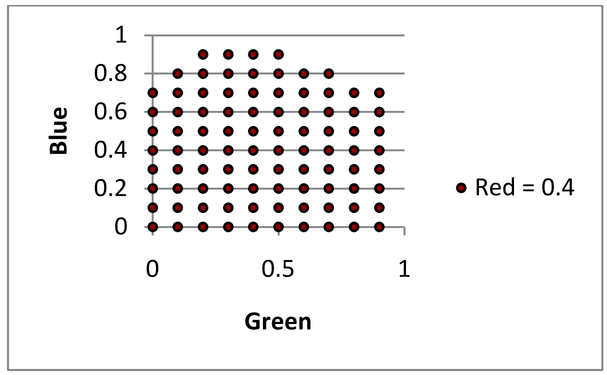
0
0.5
1
Green
Red = 0.2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0
0
0.5
1
Green
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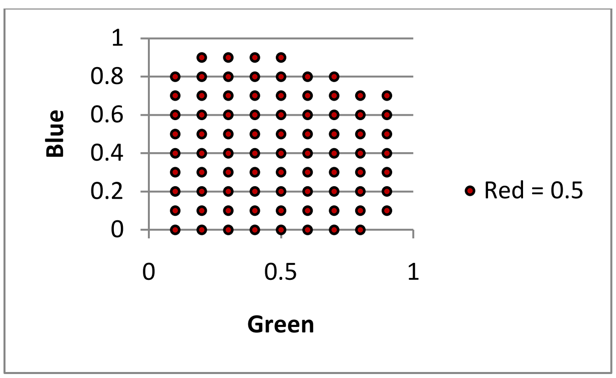
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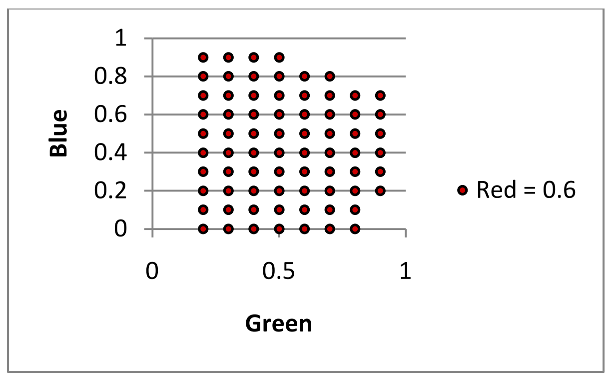
0.4
0.2
0
0
0.5
1
Green
Red = 0.5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0
0.5
1
Green
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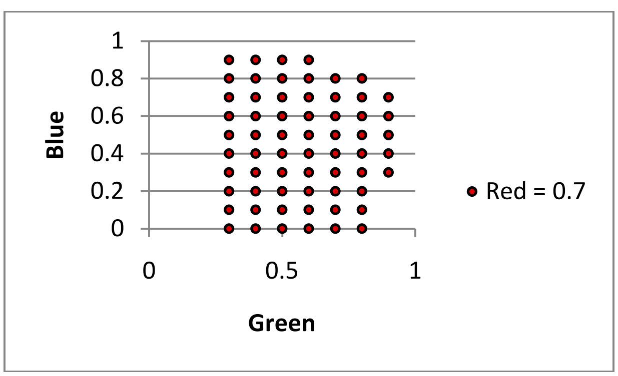
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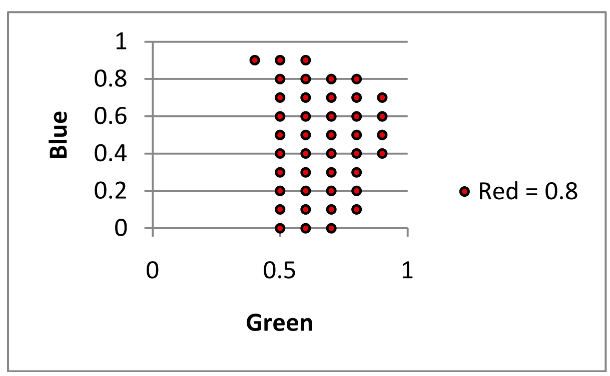
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0.2
0
0
0.5
1
Green
Red = 0.8

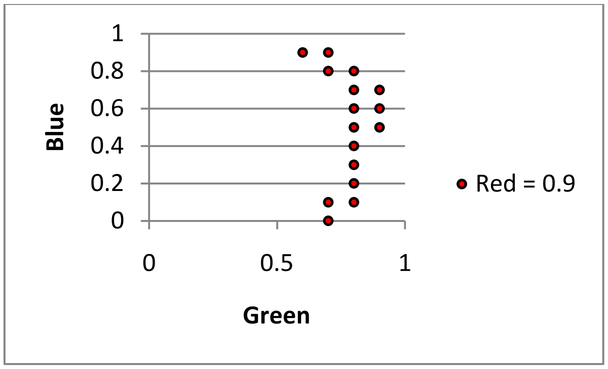

Figure 8 – Range of RGB_{709} values for $0.80 < \varepsilon < 1.25$

What these charts do not show are the worst case values. For example:

Color	R_{709}	G_{709}	B_{709}	Error
Red	0.90	0.00	0.00	0.22
Green	0.00	0.90	0.00	0.66
Blue	0.00	0.00	0.90	0.66
Cyan	0.00	0.90	0.90	3.63
Yellow	0.90	0.90	0.00	4.41
Magenta	0.90	0.00	0.90	0.39

It would be unusual to have an architectural environment where everything is painted in a single primary or complementary color, but it is still possible.

What the charts do show however is more important: the ITU-R BT.709 color space used by lighting simulations for predicting photometric quantities is acceptably accurate for most architectural environments. Fully 60 percent of the RGB_{709} color gamut yields results that are within the defined error limits.

Conclusions

Most lighting simulation programs represent color as RGB triplets in the ITU-R BT.709 color space. Implicit in this representation is the assumption that interreflections between colored surfaces can be accurately calculated using three separate color bands. We have used Sumpner's Principle and an infinite room to examine the errors due to this approximation when compared to realistic spectral reflectance distributions.

It has been shown that, apart from contrived architectural environments where everything is painted in a single primary or complementary color, the use of three color bands (red, green, and blue) in lighting calculations should yield acceptably accurate photometric quantities.

References

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